Using List Decoding to Improve the Finite-Length Performance of Sparse Regression Codes

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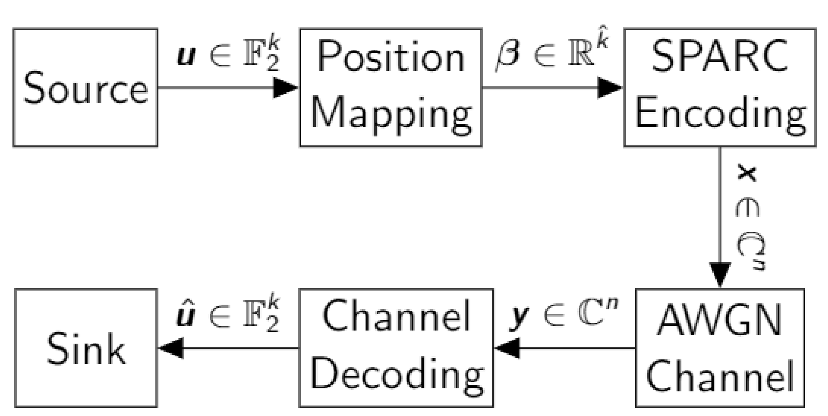
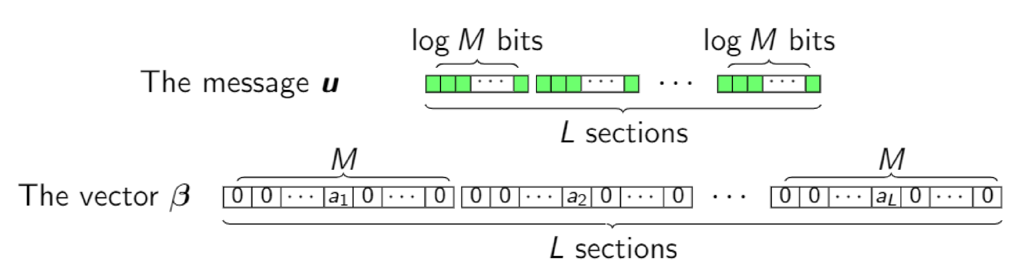
Introduction



Data Communication Model for SPARCs.

Sparse regression codes (SPARCs) were first introduced by Joseph *et al.* for efficient communication over additive white Gaussian noise (AWGN) channels. We will introduce previous works regarding SPARCs based on the above blocks.

- Position Mapping:



- SPARC Encoding: the codeword x of length n is given by the matrix-vector multiplication, i.e., $x = A\beta$.
  - ✓ Theoretically, the matrix A of size $n \times ML$ is the so-called design matrix and its entries are i.i.d. Gaussian $\sim \mathcal{CN}(0, 1/n)$.
  - ✓ Practically, use the suitably sub-sampled discrete Fourier transform (DFT) matrix.
- Various Decoders (over real-valued AWGNs): estimate β based on y, the design matrix A, and the structure of β, where y can be expressed as $A\beta + w$ and $w = (w_i)_{i\in[n]}$ with w_i i.i.d.CN$(0, \sigma^2)$ for all $i \in [n]$.
  - ✓ SPARCs were first introduced by Joseph and Barron (2012) and the optimal decoder (i.e., the maximum likelihood decoder) was proposed accordingly.
  - ✓ Joseph and Barron (2014) introduced an efficient decoding algorithm called "adaptive successive decoding".
  - ✓ An adaptive soft-decision successive decoder was proposed by Barron and Cho (2012).
  - ✓ The **approximate massage passing (AMP)** decoder was first proposed by Barbier and Krzakala (2014), and then it was rigorously proven to be asymptotically capacity-achieving by Rush *et al.*(2017).

AMP Decoding over Complex-valued AWGNs

Initialize $\beta^0 := 0$. For $t = 0, 1, 2, \ldots$, compute

$$z^t := y - A\beta^t + \underbrace{\frac{z^{t-1}}{\tau_{t-1}^2}\left(P - \frac{\beta^{t2}}{n}\right)}_{\text{“Onsager term”}},$$

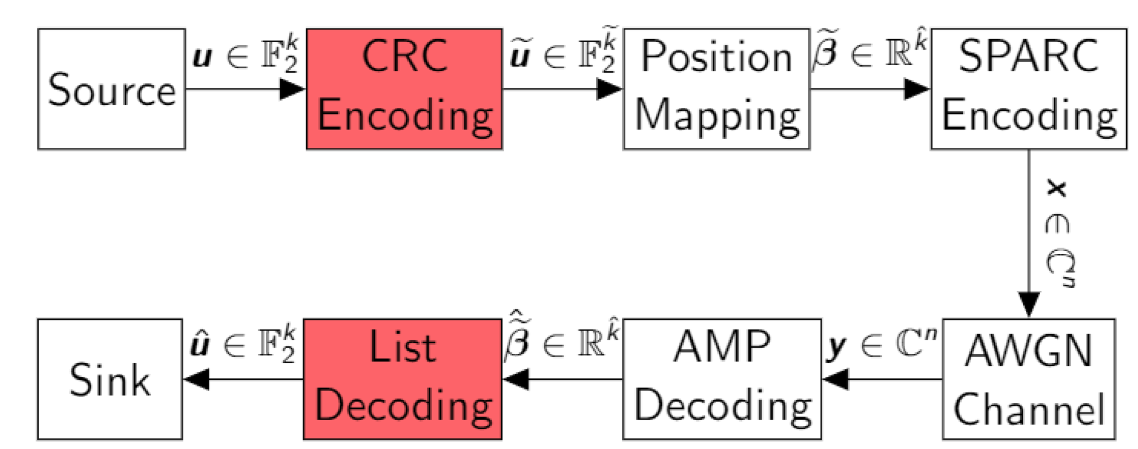
$$\beta_i^{t+1} := \eta_i^t\underbrace{\left(\beta^t + A^* z^t\right)}_{\approx \beta + \tau^t u},\quad i = 1, \ldots, ML.$$

- The additive Gaussian noise vector u has i.i.d. $\mathcal{CN}(0, 1)$ entries and is independent with β.
- The constants $\{\tau_t\}$ can be determined via the state evolution.
- In actual implementation, we use an online estimate $\hat{\tau}_t^2 = \frac{z^{t2}}{n}$.
- the denoiser functions $\eta_i^t(\cdot)$ are the Bayes-optimal estimators.

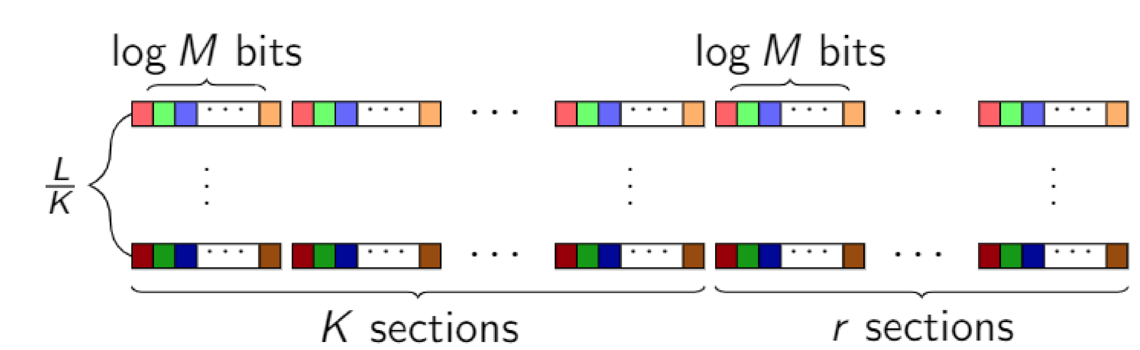
List Decoding

1. Perform T iterations of AMP decoding; the resulting estimate of $\tilde{\beta}$ is called $\tilde{\beta}^{(T)}$.
2. For each section $\ell \in [\tilde{L}]$, normalizing $\tilde{\beta}_\ell^{(T)}$ gives the a posterior distribution estimate of the location of the non-zero entry of $\tilde{\beta}_\ell$, denoted by $\hat{\tilde{\boldsymbol{\beta}}}_\ell^{(T)}$.
3. For each section $\ell \in [\tilde{L}]$, convert the posterior distribution estimate $\hat{\tilde{\boldsymbol{\beta}}}_\ell^{(T)}$ into $\log_2 M$ bit-wise posterior distribution estimates.
4. For each codeword C_i, we establish a binary tree of depth $K + r$, where, starting at the root, at each layer, we keep at most S branches, which are the most likely ones.
5. For each codeword C_i, once we have established such a binary tree, list decoding will give us S ordered candidates corresponding to the remaining S paths from the root to the leaves.

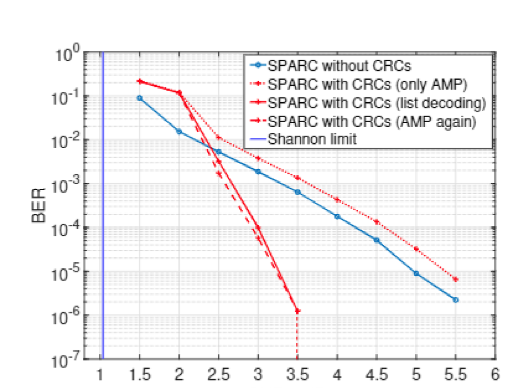
Concatenated Coding Scheme



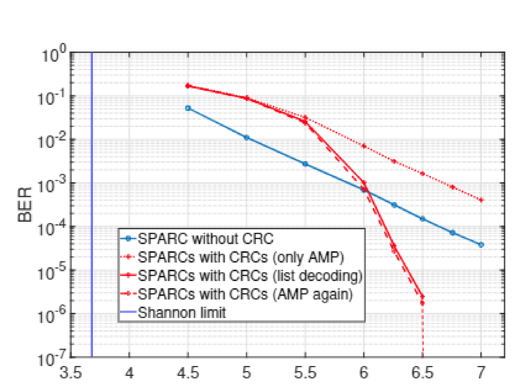
For our proposed concatenated coding scheme, we will discuss the two extra red blocks in details. The CRC Encoding will be graphically illustrated as follows.



Simulation Results



- Figure shows the BER performance comparison of low-rate SPARCs with CRC codes using list decoding and original SPARCs without CRC codes using only AMP.
- The figure shows that SPARCs concatenated with CRC codes can provide a steep waterfall-like behavior above a threshold of $\mathrm{SNR_b} = 3.5$ dB.



- Figure shows the BER performance comparison of high-rate SPARCs with CRC codes using list decoding and original SPARCs without CRC codes using only AMP.
- The figure shows that SPARCs concatenated with CRC codes can provide a steep waterfall-like behavior above a threshold of $\mathrm{SNR_b} = 6.5$ dB.

Setups for our simulation results are as follows:

- information sections $L = 1000$,
- the size of each section $M = 512$,
- CRC code information bits $K = 100$, with the generator polynomial g(x) = 0x97.

Conclusion

- We introduced AMP decoding for SPARCs over complex-valued AWGN channels.
- We proposed a concatenated coding scheme that uses SPARCs concatenated with CRC codes on the encoding side and uses list decoding on the decoding side.
- Simulation results showed that the finite-length performance is significantly improved compared with the original SPARCs.

Additional Information

The poster only discussed how to employ list decoding in SPARCs optimized by the iterative power allocation scheme; there are lots of interest directions for future work, and we name a few as follows.

- Apply this concatenated coding scheme to spatially-coupled SPARCs.
- Give an (information) theoretical analysis of our proposed list decoding scheme.
- Suitably fit this concatenated coding scheme in unsourced random access scenario.

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